Computational Models, Spring 2015 Exrecise #2

Non-Regular Languages and Context free grammars

1. We showed in class that Regular languages are closed under union. Are they closed under infinite union? Prove your answer formally.

2. Let L be a regular language state for each language if it is regular or not. Give a complete formal proof:

   (a) $L_a = \{ww' : w \in L \text{ and } w' \notin L\}$

   (b) $L_b = \{y : \exists x, z \ s.t. |x| = |z| \text{ and } xyz \in L\}$

   (c) $L_c = \{xy : |x| = |y| \text{ and } \exists \sigma \ s.t. \ x\sigma y \in L\}$

   (d) $\text{Pref}(L) = \{x : \exists y \ s.t. \ xy \in L\}$

3. Prove that the following languages are not regular

   (a) $L_a = \{(ab)^n c^n : n \geq 0\}$ above $\Sigma = \{a, b, c\}$

   (b) $L_b = \{a^i b^j c^k : i = j \text{ or } j = k\}$ above $\Sigma = \{a, b, c\}$

   (c) $L_c = \{ww : w \in \{0, 1\}^*\}$

   (d) $L_d = \{\text{balanced strings }\}$ above $\Sigma = \{(,)\}$
   A string is balanced if the number of (equals the number of) and while reading the string, the number of (is equal or greater than the number of).

   (e) $L_e = \{\{0\}^n \mid n \text{ is a prime number}\}$ above $\Sigma = \{0\}$

4. Prove that the following languages are not regular using the Myhill-Nerode Theorem

   (a)
   $$L = \{ww \mid w \in \Sigma^*\} \text{ for any } \Sigma^* \text{ such that } |\Sigma| > 1$$

   (b) Let L be the set of algebraic expressions involving identifiers x and y, operations $+$ and $*$ and left and right parentheses. L can be defined recursively as follows:
   Basis Clause: x and y are in L.
   Inductive Clause: If α and β are in L, then $(\alpha + \beta)$ and $(\alpha * \beta)$ are in L.
   Extremal Clause: Nothing is in L unless it is obtained from the above two clauses.

   For example, x , $(x * y)$, $((x + y) * x)$ and $(((x * y) + x) + (y * y))$ are algebraic expressions.

5. Give an example of a non-regular language L and a homomorphism h such that $h(L)$ is regular.

6. Let $L = L((00 + 1)^*)$ and $h : \{a, b\}^* \to \{0, 1\}^*$ be defined by $h(a) = 01$ and $h(b) = 10$.
   What is $h^{-1}(L)$? Prove your answer formally.
   Hint: You need to prove that $h(w) \in L$ iff $w = (?)^n$ where ? is some string above $\{a, b\}$. One direction (if...) is easy. For the other direction (only-if) assume that $h(w)$ is in L and shoe that it is in the right form. This can be done by considering all the possible conditions where a string is NOT of that form and showing that if any of them hold, then $h(w)$ is in not L

7. Given the following grammar,
   $$S \to PS|\varepsilon \qquad P \to (S)$$
   Show how to construct the word (())(). Explain (in words) what is the language described by this grammar.