Exercise 1 - Computational Models - Spring 2015

Notation: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

1. For each of the following languages over $\Sigma = \{0, 1\}$, present a drawing representing a DFA that accepts it and a formal definition of the DFA (correctness proof not needed):

   (a) Σ^*

   (b) $\{\varepsilon, 1, 01\}$

   (c) $\{w \mid w \text{ does not contain } 0 \text{ or } w \text{ ends with } 01\}$

   (d) $\{w \mid \#_0(w) \bmod 3 = \#_1(w) \bmod 3\}$

2. For the language $L = \{w01 \mid w \in \Sigma^*\}$ over $\Sigma = \{0, 1\}$ (i.e. the collection of all words ending with 01), present a drawing representing a DFA that accepts it and a formal definition. Prove correctness.

3. Present an NFA (drawing and formal definition) and convert it to a DFA for the following languages over $\Sigma = \{0, 1\}$:

   (a) $\{w| \ w \text{ contains } '00' \text{ or doesn't contain } '101'\}$

   (b) $\{xy| \ \#_0(x) \bmod 2 = 0 \text{ and } \#_1(y) \bmod 2 = 1\}$

4. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:

   (a) $\{w| \ |w| \bmod 4 = 0\}$

   (b) $\{w| \ w \text{ contains exactly three '1's}\}$

   (c) The complement of $\mathcal{L}((1 \cup 01 \cup 001)^*(\varepsilon \cup 0 \cup 00))$

5. Given that L is a regular language over some alphabet Σ, prove that the following languages are regular:

   (a) $\{xy \mid x \in L, \ y \notin L\}$

   (b) $\{x_1x_2 \cdots x_k \mid x_1, \ldots, x_k \in \Sigma \text{ and } \exists y_1, y_2, \cdots, y_k \in \Sigma, x_1y_1x_2y_2 \cdots x_ky_k \in L\}$