Exercise 4 - Computational Models - Spring 2015

1. Prove or disprove:
   (a) $\mathcal{RE}$ is closed under complementation.
   (b) $\mathcal{RE}$ is closed under intersection.
   (c) co-$\mathcal{RE}$ is closed under intersection.

2. In this question you are asked to give a formal description of a TM, i.e. describe formally each one of the elements in $M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$.
   (a) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Give a formal description of a TM that accepts $L(A)$
   (b) Let $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA. Give a formal description of a TM that accepts $L(P)$

3. Let $\text{Prefix}(L) = \{x \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$. Prove that $\mathcal{RE}$ is closed under Prefix.

4. A function $f : \Sigma^* \to \Gamma^*$ is computable if there exists a TM that for every $x \in \Sigma^*$ halts with $f(x)$ on its tape, when given x as input. For a function $f : \Sigma^* \to \Gamma^*$ let language $L_f = \{(x, f(x)) | x \in \Sigma^*\}$.

   Show that $L_f \in \mathcal{RE} \iff f$ is computable.

5. Prove or disprove:
   (a) If $L_1, L_2 \in \mathcal{R}$ and $L_1 \subseteq L \subseteq L_2$ then $L \in \mathcal{R}$.
   (b) If $L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$ then $L_1 \cap L_2 \notin \mathcal{R}$.
   (c) If $L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$ then $L_1 \cup L_2 \notin \mathcal{R}$.
   (d) $\nexists L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$ s.t. $L_1 \cup L_2 \in \mathcal{R}$ and $L_1 \cap L_2 \in \mathcal{R}$.