Computational Models, Spring 2015 Exrecise #5

Reductions

1. Show that the following language is undecidable:

$$L_{01} = \{< M > \ \mid M \text{is a TM that accepts the string } 01\}$$

2. Prove or show a counter-example: If $A \leq_M B$ and B is a regular language, does that imply that A is a regular language?

3. A useless state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a state in a Turing machine is useless:

$$\text{USELESS}_{TM} = \{< M >, q \mid q \text{ is a useless state in the TM } M\}$$

   Show that USELESS_{TM} is undecidable.

4. Show that S_{TM} is undecidable, where

$$S_{TM} = \{< M > \ \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$$

5. Fermats Last Theorem, until recently one of the most famous unproved statements in mathematics, asserts that there are no integer solutions (x, y, z, n) to the equation $x^n + y^n = z^n$ satisfying $x, y > 0$ and $n > 2$. Show that if the halting problem would be decideable it would allow you to determine whether the statement is true or false.

6. Show that the problem of deciding whether hamsters are smarter than humans is decidable. More precisely, show that there is a TM that will print "YES" if hamsters are smarter than humans and "NO" otherwise.

7. For the language below, determine whether or not it is in $\mathcal{R}$. If it is in $\mathcal{R}$, describe a Turing machine that decides it. If it is not, show this using a reduction.

$$L = \{< M > \ \mid M \text{ is a TM that decides the halting problem}\}$$