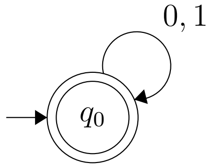
Solution sketch 1 - Computational Models - Spring 2015

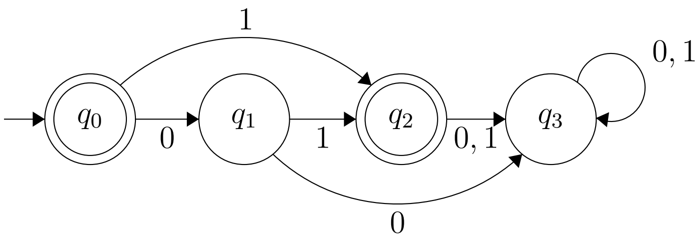
1. (a) Σ^*



$A = (Q = \{q_0\}, \Sigma = \{0,1\}, \delta, q_0, \{q_0\})$, δ :

	0	1
q_0	q_0	q_0

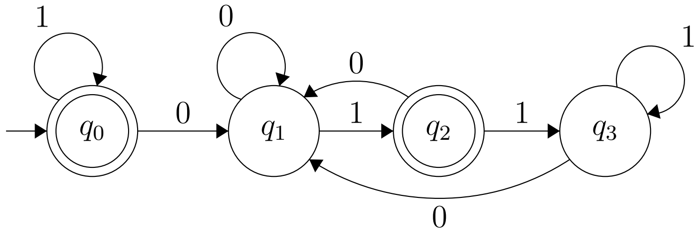
(b) $\{\varepsilon, 1, 01\}$



$A = (Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{0,1\}, \delta, q_0, \{q_0, q_2\})$, δ :

	0	1
q_0	q_1	q_2
q_1	q_3	q_2
q_2	q_3	q_3
q_3	q_3	q_3

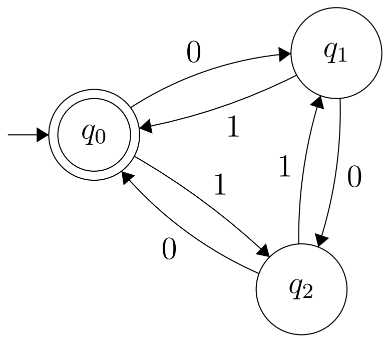
(c) $\{w \mid w \text{ does not contain } 0 \text{ or } w \text{ ends with } 01\}$



$A = (Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{0,1\}, \delta, q_0, \{q_0, q_2\}),\ \delta :$

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_1	q_3

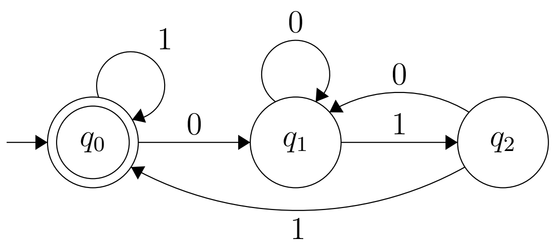
(d) $\{w \mid \#_0(w) \bmod 3 = \#_1(w) \bmod 3\}$



$A = (Q = \{q_0, q_1, q_2\}, \Sigma = \{0,1\}, \delta, q_0, \{q_0\}),\ \delta :$

	0	1
q_0	q_1	q_2
q_1	q_2	q_0
q_2	q_0	q_1

2. $L = \{w01 \mid w \in \Sigma^*\}$



$A = (Q = \{q_0, q_1, q_2\}, \Sigma = \{0,1\}, \delta, q_0, \{q_2\}),\ \delta :$

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_0

To prove that $\mathcal{L}(A) = L$, you need to prove by indiction on word length the following claim (as seen in class):

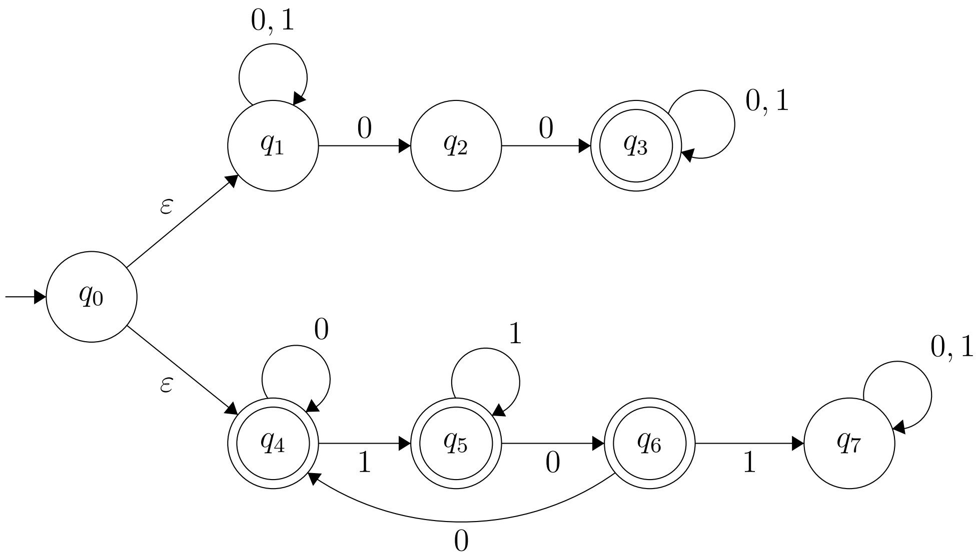
Claim. *Let* $\mathcal{L}'_i = \{x \in \Sigma^* \mid \widehat{\delta}(q_0, x) = q_i\}$ *for every* $i \in \{0,1,2\}$ *and let*

- $\mathcal{L}_0 = \{w11 \mid w \in \Sigma^*\} \cup \{\varepsilon, 1\}$
- $\mathcal{L}_1 = \{w0 \mid w \in \Sigma^*\}$

- $\mathcal{L}_2 = \{w01 \mid w \in \Sigma^*\}$

Then $\mathcal{L}'_i = \mathcal{L}_i$ *for every* $i \in \{0, 1, 2\}$

3. (a) $\{w \mid w$ contains $'00'$ or doesn't contain $'101'\}$

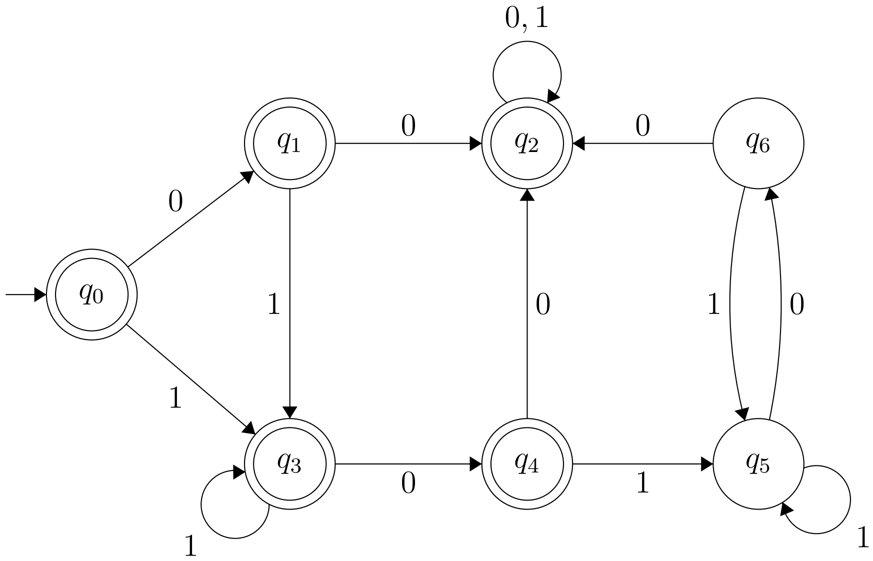


$N = (Q = \{q_0, \ldots, q_7\}, \Sigma = \{0, 1\}, \delta, \{q_0\}, \{q_3, \ldots, q_6\})$,

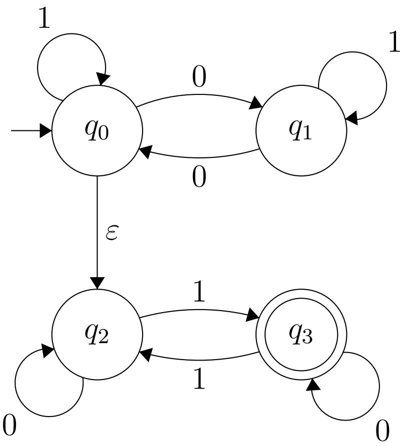
δ :

	0	1	ε
q_0	$\emptyset$	$\emptyset$	$\{q_1, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\emptyset$
q_3	$\{q_3\}$	$\{q_3\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_5\}$	$\emptyset$
q_5	$\{q_6\}$	$\{q_5\}$	$\emptyset$
q_6	$\{q_4\}$	$\{q_7\}$	$\emptyset$
q_7	$\{q_7\}$	$\{q_7\}$	$\emptyset$

An equivalent DFA (after simplifications)



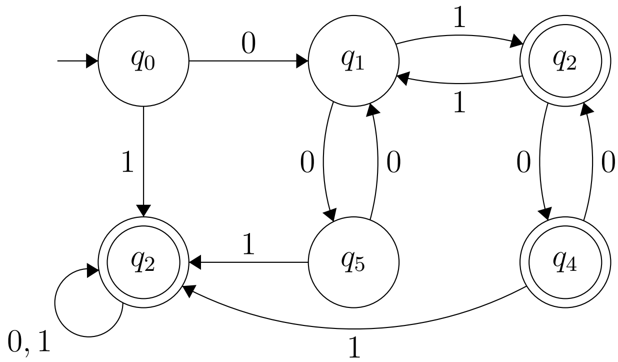
(b) $\{xy |\ \#_0(x) \bmod 2 = 0 \text{ and } \#_1(y) \bmod 2 = 1\}$



$N = (Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{0, 1\}, \delta, \{q_0\}, \{q_3\})$, δ :

	0	1	ε
q_0	$\{q_1\}$	$\{q_0\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$	$\emptyset$
q_2	$\{q_2\}$	$\{q_3\}$	$\emptyset$
q_3	$\{q_3\}$	$\{q_2\}$	$\emptyset$

An equivalent DFA (after simplifications)



4. (a) $((0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1))^*$
   (b) $0^*10^*10^*10^*$
   (c) $(0 \cup 1)^*000(0 \cup 1)^*$

5. (a) This language is $L\bar{L}$. Since L is regular and the regular languages are closed under the operators complement and concatenation, the language at hand is also regular.
   (b) Let the desired language be L'. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting L. We construct an NFA, $N = (Q, \Sigma, \delta', q_0, F)$, where $\forall q \in Q, \sigma \in \Sigma, \delta'(q, \sigma) = \{\delta(\delta(q, \sigma), a) : a \in \Sigma)$. Thus, we execute two steps in A: one with respect to to the character σ and one that we "guess" and can use any character in Σ. You should prove that $L(N) = L'$. The key observation is that $\delta^*(q_0, w = x_1y_1...x_ny_n) = q$ (i.e. running w in A terminates in state q) iff there is a computation in N for $x_1x_2...x_n$ that ends in q.